Notes - Rationalizing

Recall:

- Rational numbers $\mathbb{Q}$ can be expressed as a fraction $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Its decimal form terminates or repeats. Ex: $\frac{1}{2}$, 0.68, $\sqrt{9}$, $0.\overline{3}$
- Irrational $\mathbb{I}$ - not rational. Decimal doesn't terminate or repeat Ex: π, $\sqrt{2}$

Key Ideas:

1. "Well Chosen 1" ("WC1")

   $\frac{a}{a} = 1$ Anything divided by itself = 1

   $x \cdot 1 = x$ Anything $\times 1$ is itself (1 is the multiplicative identity)

2. $\sqrt{c} \cdot \sqrt{c} = c$ $\quad (\sqrt{c})^2 \quad \sqrt{c^2}$

   ⇒ called "rationalizing" b/c it turns $\mathbb{I}$ into $\mathbb{Q}$

RATIONALIZE (the denominator)

Ex 1: $\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$

"WC1"

Ex 2: $\frac{3}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{2 \cdot 5} = \boxed{\frac{3\sqrt{5}}{10}}$

Ex 3: Conjugate - only change the sign in the middle

- When your denominator has $a \pm \sqrt{b}$, your "WC1" is the conjugate

$$\frac{7}{4-\sqrt{3}} \cdot \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{7(4+\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})} = \frac{28+7\sqrt{3}}{16 + \cancel{4\sqrt{3}} - \cancel{4\sqrt{3}} - 3} = \boxed{\frac{28+7\sqrt{3}}{13}}$$

conjugate

FOIL
Box
Double - Distribute